Artificial Neural Network Modeling of Thermal Conductivity Changes in Milk during Mechanized Khoa Production

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Received:- 10 April 2022/ Revised:- 17 April 2022/ Accepted:- 26 April 2022/ Published: 30-04-2022

Abstract— *An artificial neural network (ANN) approach was successfully deployed to model and predict the thermal conductivity of milk and concentrated milk systems during mechanized khoa production. Reliable data points (n=203) spanning wide operational boundaries of temperature (275.15–353.10 K), moisture content (48.82–92.00%), and fat content (0.00–11.17%) were compiled from established experimental studies to formulate and validate the model. A multi-layer feed-forward network optimized via the quasi-Newton algorithm using a 3:3:1 topology (three inputs, three hidden neurons with hyperbolic tangent activation functions, and a single linear output layer) demonstrated optimal predictive behaviour. The architecture yielded outstanding precision on independent testing subsets, demonstrating a strong correlation coefficient (R = 0.986), a minimal root mean squared error (RMSE = 0.0084 W/m•K), and a normalized squared error of 0.029 (normalized to the variance of the target data). Input sensitivity computations verified that product temperature (31.4% contribution) and moisture content (30.2% contribution) exert the highest thermodynamic control on thermal conductivity shifts, whereas fat content (4.4% contribution) exhibits a weaker but consistently inverse linear relationship. The resolved predictive equations were effectively embedded within a highly practical Microsoft Excel-based graphical user interface (GUI) to assist dairy process designers in real-time calculation, simulation, and industrial scaling of continuous scraped surface heat exchangers for indigenous milk confectionery manufacturing.*

Keywords— Artificial Neural Network, Thermal Conductivity, Milk Desiccation, Khoa, Process Optimization, Scraped Surface Heat Exchanger.

I. INTRODUCTION

Khoa is one of the most prominent traditional indigenous dairy products in the Indian subcontinent. It is conventionally manufactured by the gradual heating, desiccation, and continuous concentration of whole milk in open kettles at atmospheric pressure, combined with continuous manual scraping and stirring until a dense, semi-solid dough-like consistency is achieved. Industrially, khoa serves as an essential intermediate base matrix for an extensive portfolio of traditional sweets including peda, burfi, milk cake, kalakand, and gulabjamun. Annually, approximately 600,000 metric tons of khoa are manufactured within India alone, representing a vital utilization channel for nearly 7% of the nation's total fluid milk production.

Driven by the growing urban and commercial demand for uniform-quality milk sweets, the dairy sector has progressively moved away from batch-oriented cottage-level preparation toward large-scale mechanized desiccation systems. Modern continuous machinery — specifically single and multi-stage inclined scraped surface heat exchangers (SSHE) and thin-film evaporation plants — has been developed to support industrial throughput. However, raw milk and its concentrated intermediates display non-linear, non-Newtonian behaviour and are highly susceptible to chemical degradation, thermal discoloration, and fouling if process heat flux is inappropriately regulated.

The rational design, computer-aided simulation, and precise control of modern high-efficiency heat exchange equipment depend fundamentally on accurate knowledge of the physical and thermodynamic properties of the milk matrix across the entire path of its concentration gradient. Among these properties, thermal conductivity (k) stands out as a critical parameter driving transient heat transfer and temperature distributions within the product film. Despite its obvious importance, historical data regarding the thermal conductivity of milk are often constrained to narrow temperature or solids ranges, typically treated as simple binary water-solid systems or presented solely in fragmented tabular or graphical formats that are cumbersome for continuous spreadsheet-based engineering calculations.

Furthermore, thermal conductivity is heavily influenced not only by gross moisture content but also by complex structural rearrangements among the constituent solids, including lipid state transformations and protein concentration cross-linking under elevated temperature kinetics. Because milk composition varies continuously across the multi-stage desiccation cycle of khoa, empirically measuring thermal conductivity across all prospective micro-states is practically unfeasible. Consequently, a reliable and mathematically robust predictive framework is required.

Artificial Neural Networks (ANN) represent a powerful class of data-driven computing models capable of learning intricate non-linear relationships directly from numerical examples without demanding precise prior physical formulations. ANNs excel at managing structural uncertainties and noisy experimental measurements. This study aims to develop, optimize, and deploy a compact, highly precise feed-forward ANN framework capable of accurately mapping thermal conductivity as a direct function of product temperature, moisture content, and fat content, thereby providing process engineers with an accessible, high-fidelity modelling tool for traditional dairy process scaling.

II. MATERIALS AND METHODS

2.1 Data Sourcing and Standardization:

To build a robust and statistically sound training foundation, a comprehensive dataset consisting of 203 high-resolution experimental observations was compiled from authoritative literature. The primary sources included Pereira et al. (2013), Fernández-Martín and Montes (1972), and Minim et al. (2002). For publications where experimental findings were presented exclusively in graphical formats, a validated, high-precision HTML5 web-based plot digitizer (WebPlotDigitizer, Version 4.0) was utilized to extract precise numerical values from coordinate graphs. All source data were standardized into uniform SI units: Temperature (T) in Kelvin (K), Moisture Content (M) in percent (%), Fat Content (F) in percent (%), and effective Thermal Conductivity (k) in W/m•K, with all records maintained to three decimal places inside a comma-separated values (.csv) master file. No additional data transformation, interpolation, or outlier removal was performed beyond unit standardization and digitization.

2.2 Neural Network Architecture and Software Deployment

The predictive modelling workflow was executed using the industrial-grade data analytics suite Neural Designer (Artificial Intelligence Techniques, S.L.). The structural dataset was parsed into three distinct subsets via randomized partitioning using a fixed random seed (set to 42 for reproducibility): 123 instances (60.6%) dedicated strictly to model parameter training, 40 instances (19.7%) allocated for model order selection to avoid overfitting, and the remaining 40 instances (19.7%) preserved as an independent testing set to execute rigorous validation metrics. The testing set was partitioned before any hyperparameter tuning or model selection to ensure genuine independence.

TABLE 1
STATISTICAL BOUNDARIES AND VARIANCE PARAMETERS OF THE COMPREHENSIVE MODELLING DATASET

Variable Axis	Minimum Boundary	Maximum Boundary	Mean Value	Standard Deviation
Temperature (K)	275.15	353.1	313.356	23.429
Moisture Content (%)	48.82	92	81.33	10.825
Fat Content (%)	0	11.17	4.076	2.826
Thermal Conductivity (W/m•K)	0.431	0.652	0.566	0.053

Notably, the dataset's moisture range (48.82–92.00%) adequately captures the entire khoa production spectrum, including final product moisture targets of approximately 50–55%.

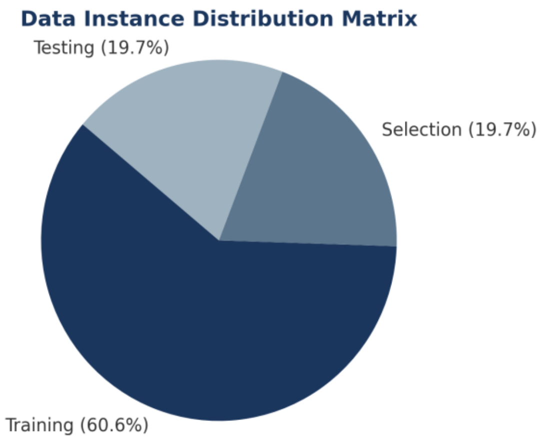

FIGURE 1: Schematic partitioning allocation for the modelling dataset

A systematic hyperparameter grid search involving 12 complete modelling iterations was conducted. Variables evaluated included data scaling profiles, hidden neuron layer depth, activation functional selection, and regularized loss formulations. To establish the predictive framework, an automated input scaling layer was coupled with a 3:3:1 multilayer perceptron scheme. Layer 1 combined the three input nodes with a hidden layer containing three perceptron units governed by non-linear hyperbolic tangent (tanh) transfer functions, which mapped into Layer 2 — a single-neuron linear output assembly that generated the final unscaled thermal conductivity value. Training was terminated when the selection subset error failed to decrease for five consecutive epochs, preventing overfitting.

III. RESULTS AND DISCUSSION

3.1 Linear Correlation Matrix and Preliminary Insights

An initial evaluation of linear dependencies between single input drivers and the target thermal conductivity was conducted by computing absolute Pearson correlation coefficients. The target thermal conductivity demonstrated its strongest direct linear relationship with moisture content (correlation coefficient = 0.757) and temperature (0.673), while displaying a clear inverse linear trend against fat content (-0.576). This confirms that as desiccation progresses (moisture decreases and fat concentration increases), the effective thermal conductivity of the concentrated milk solution falls substantially, expanding the required surface area or thermal residence time within the scraped surface heat exchange cylinders. These linear correlations provide initial insights but do not capture the full non-linear interactions among variables, justifying the ANN approach adopted in this study.

3.2 Network Training and Input Sensitivity Analysis

The network parameters were optimized using the quasi-Newton training algorithm, which provided rapid convergence without requiring the computation of complex second-order partial derivatives. Post-convergence, input importance metrics were derived by evaluating selection loss changes when removing individual input variables sequentially.

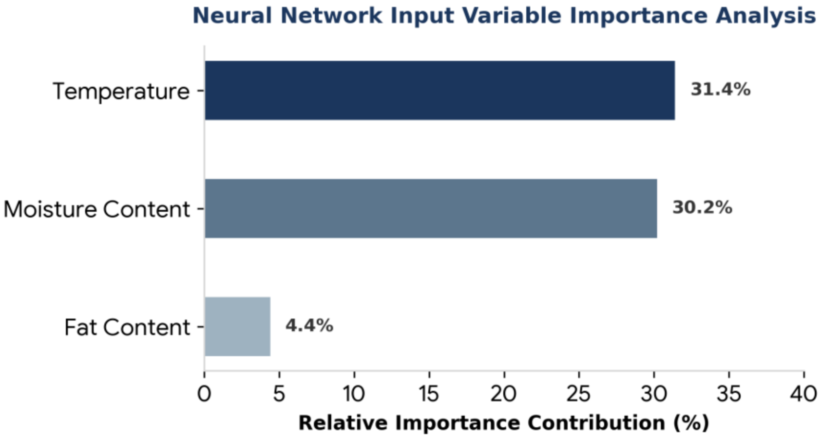

FIGURE 2: Relative importance profile of input variables on thermal conductivity prediction

As illustrated in Figure 2, product temperature emerged as the primary variable with a relative performance contribution of 31.4%, closely followed by moisture content at 30.2%. Fat content demonstrated a lower relative importance of 4.4%. The remaining variance (34.0%) is attributable to non-linear interaction effects among the three input variables, which the ANN captures but cannot be uniquely assigned to individual inputs. This reveals that while moisture and temperature dictate macro-level kinetic energy transmission through the aqueous phase, lipid composition acts as a secondary structural factor that modifies the overall heat transfer profile.

3.3 Model Performance Validation and Error Metrics

The finalized 3:3:1 network was validated against an independent testing subset. The model demonstrated excellent agreement, achieving a testing root mean squared error (RMSE) of 0.0084 W/m•K and a normalized squared error of 0.0294. A linear regression analysis relating the scaled model outputs to the target testing records yielded a y-intercept of 0.00822, a slope of 0.954, and a correlation coefficient (R) of 0.986, confirming high predictive accuracy.

TABLE 2
COMPREHENSIVE TRAINING, SELECTION, AND TESTING ERROR MATRIX

Error Metric Formulation	Training Subset (n=123)	Selection Subset (n=40)	Testing Subset (n=40)
Sum Squared Error (SSE)	0.01016	0.00137	0.00282
Mean Squared Error (MSE)	8.26E-05	3.42E-05	7.06E-05
Root Mean Squared Error (RMSE)	0.00909	0.00585	0.0084
Normalized Squared Error (NSE)*	0.02968	0.01211	0.02948

NSE = MSE / variance of target values, providing a scale-independent measure of model accuracy.

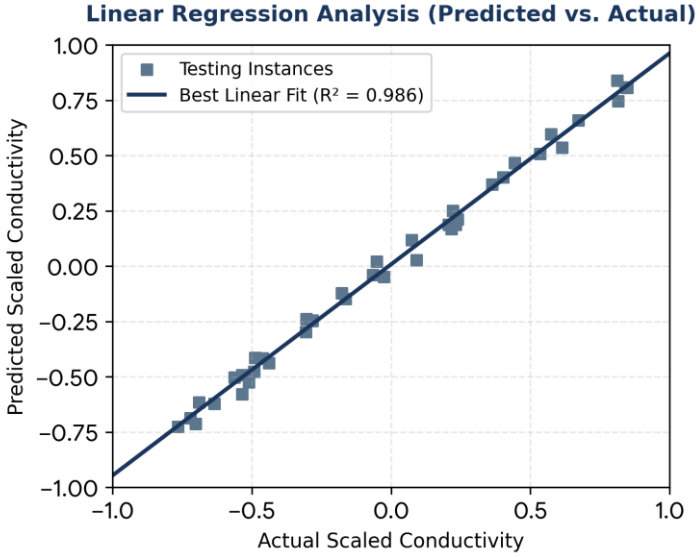

FIGURE 3: Linear regression analysis of predicted vs. actual scaled thermal conductivity values

3.4 Final Explicit Mathematical Formulation

To support direct integration into process engineering software or automation logic, the neural network's internal weights and biases were extracted and compiled into an explicit sequence of algebraic expressions. Given the inputs Temperature (K), Moisture (%), and Fat (%), the effective thermal conductivity is determined using the following mathematical equations:

Step 1: Input Scaling Realignment

*scaled_Temperature = 2 * (Temperature - 275.15) / (353.1 - 275.15) - 1*

scaled_Moisture = (Moisture - 81.3298) / 10.8246

scaled_Fat = (Fat - 4.07557) / 2.82601

Note: Scaling parameters are presented at the precision output by Neural Designer; the corresponding mean and standard deviation values from Table 1 (81.330 and 10.825 for moisture; 4.076 and 2.826 for fat) may be used with minimal loss of accuracy.

Step 2: Hidden Layer Perceptron Activations

y_1_1 = tanh(-0.352987 + 0.224189 × scaled_Temperature + 0.0283239 × scaled_Moisture - 0.213047 × scaled_Fat)

y_1_2 = tanh(1.250900 + 0.592517 × scaled_Temperature - 0.115882 × scaled_Moisture - 0.0698833 × scaled_Fat)

y_1_3 = tanh(-0.417119 - 0.133327 × scaled_Temperature + 0.535689 × scaled_Moisture + 0.390007 × scaled_Fat)

Step 3: Linear Synthesis and Output Unscaling

scaled_Conductivity = -0.177078 + 1.51613 × y_1_1 + 1.43764 × y_1_2 + 0.818709 × y_1_3

Conductivity (W/m•K) = 0.5 × (scaled_Conductivity + 1.0) × (0.652 - 0.431) + 0.431

3.5 Graphical User Interface (GUI) Deployment:

To translate these mathematical expressions into a highly accessible tool for dairy field engineers, a graphical user interface was developed in Microsoft Excel. This interface features entry blocks for Temperature, Moisture, and Fat, which feed directly into a cell-mapped execution script of the model equations to display real-time thermal conductivity outputs. For example, inputting a process state of Temperature = 333.15 K (60°C), Moisture = 84.00%, and Fat = 7.71% yields a thermal conductivity prediction of 0.549 W/m•K. This value aligns closely with experimental observations, demonstrating the model's utility for sizing and optimizing continuous heat exchangers across typical processing temperature ranges.

IV. SUMMARY AND CONCLUSION

This study successfully developed and validated a compact 3:3:1 feed-forward artificial neural network optimized via the quasi-Newton algorithm to predict the thermal conductivity of milk during khoa production. The developed network achieved high predictive accuracy, demonstrating a strong testing correlation coefficient of 0.986 and a minimal RMSE of 0.0084 W/m•K across diverse operational ranges. Input sensitivity analysis revealed that product temperature and moisture content exert primary thermodynamic control over the system (31.4% and 30.2% contributions, respectively), while fat content serves as a secondary, inversely correlated structural variable (4.4% contribution), with the remaining variance attributable to non-linear interaction effects among the three inputs. By converting the trained network into explicit algebraic equations and embedding them within an accessible Microsoft Excel GUI, this research provides a practical, high-fidelity design tool. This framework can assist dairy process engineers in accurately simulating, sizing, and controlling automated heat exchange equipment, helping transition traditional indigenous dairy confectionery manufacturing toward efficient, modern industrial scales.

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