# The Substitution of Cereal Production Factors: DEA-based Approach

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***Abstract*—** *The elasticities of substitution are derived from the optimal slack variables obtained from the additive models which are based on the DEA method. Measuring the substitutability of cereal production factors in the study area allows us to see the existence of substitutability between the land and other production factors as well as between mechanization and other inputs except the hands artwork. The estimated elasticities of substitution between irrigation water and other inputs allow us to conclude that the irrigation water and labor are substitutable.*

***Keywords*— *Nonparametric approach, efficiency borders, marginal rate of substitution, elasticities of substitution.***

## I. INTRODUCTION

The demand and supply in the Tunisian cereal market is not balanced which pushes the state to import the quantities needed to restore the balance between production and demand for cereals. The cereal coverage rate, however, is linked to the limited production and to the changes in the consumption of cereals.

Public authorities' goal is to improve the efficiency of the grain industry in order to increase the budgetary expenditure of the state. For more efficiency, the state budget, among other things, reduces the production cost. This reduction of costs in this sector requires improving the performance of cereal farms. The study of the effectiveness of cereal farms raises the issue of the use of resources in production. Efficiency is achieved by the best use of production factors. The production function describes the best practices found in the production boundary. Indeed, by implementing the same technology, differences may exist when using production factors between different cereal farms. In 1951, Debreu invented the first measurement of the technical efficiency of farms by a coefficient of resource utilization. Then, Farrell (1957) provided a measure of the efficiency comprising two components: technical efficiency and allocative efficiency. The first measures the distance between the operation and the end product. The second describes the necessary adjustments in the production factors in order for a technically efficient operation to minimize its cost.

If improving the quantity of goods produced is impossible without increasing the amount of one or more factors of production, then the operation will be technically efficient. Achieving this technical efficiency does not represent the best choice of production because there may exist some other combination of inputs in the production frontier which can reduce costs. The allocative efficiency is represented by the movement of a technically efficient point to another one on the border along the production frontier where production costs are minimized. This displacement causes a substitution of certain factors. Low allocative efficiency means that the operation uses a very low input ratio compared to one that minimizes production costs. After reaching the technical efficiency, allocative efficiency must perform a significant substitution between its inputs to improve.

The objective of this work is to determine the possibilities of substitution among Tunisian cereal production factors using nonparametric approach through the additive model. The additive model and the measure of the marginal rates of substitution are presented in the first section, while the application of this model to the cereal technology and the interpretation of results are presented in the second section.

## II. ADDITIVE MODEL AND MEASURE THE MARGINAL RATE OF SUBSTITUTION

In this part, the additive model is introduced to decompose and calculate economic efficiency. Then, this model is used as a tool to calculate the economic relationships that we need such as the marginal rates of substitution.

Cooper Park and Pastor (1999) use additive models to decompose economic efficiency into allocative and technical efficiency. This decomposition is deduced form additive approaches which have been used for Farrell (1957). However, our goal is to further investigate this and show how to calculate the marginal rate and substitution elasticities and other relationships based on these additive models.

In the DEA (Data Envelopment Analysis) literature, the economic efficiency can be distinguished by the combination between technical efficiency and allocative efficiency. These relationships can also be established in additive form, as shown in Cooper Park and Pastor (1999). This study will use the following version of the additive model (ADD) which was adopted by Cooper Park and Pastor to measure economic efficiency.

$$\max \sum_{r=1}^{s} p_r s_{r0}^{+} + \sum_{i=1}^{m} c_i s_{i0}^{-}$$
$$\begin{cases} y_{r0} = \sum_{j=1}^{n} y_{rj}\gamma_j - s_{r0}^{+} \quad r = 1 \ldots\ldots s \\ \text{S/c} \quad x_{i0} = \sum_{j=1}^{n} x_{ij}\gamma_j + s_{i0}^{-} \quad i = 1 \ldots\ldots m \\ 1 = \sum_{j=1}^{n} \gamma_j \\ \gamma_j \geq 0 \end{cases} \quad (1)$$

The outputs and inputs for decision making units, coded as DMUj, j = 1.... n, are respectively represented by $Y_{rj}$, $X_{ij}$. The $\gamma_j$ are forced to be non-negative. However, the additive model allows slack variables (input and output slack) to be either negative or positive. We assume unit prices $p_r$ for outputs slack $(s_{r0}^{+})$ and unit cost $c_i$ for the inputs slack $(s_{i0}^{-})$. This model can be mathematically linked to the models of technical and allocative efficiency in the following form:

Economic efficiency = technical efficiency + allocative efficiency (a)

As shown in Cooper Park and Pastor, this formulation can be used to provide a measure of maximum profit. It is defined by Fare, Grosskopf and Lovell (1985) as follows:

Maximum profit = real profit + maximum overall efficiency (b)

The dual problem to the above (1) can be expressed as follows:

$$\min \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_{ri} y_{r0} + u_0$$
$$\begin{cases} \text{S/c} \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_{ri} y_{r0} + u_0 \geq 0 \quad \forall j = 1 \ldots m \\ v_i \geq c_i \text{ et } u_r \geq p_r \end{cases} \quad (2)$$

To understand this model we refer to Figure 1 where each of the four production unit A, B, C and D has an output and an input. Since the additive model which has the same production possibilities as BCC, the production frontier in our case is represented by the two segments AB and BC. And since the production unit D is not effective, moving from this unit to the border is done with $s^+$ and $s^-$ as it is indicated by the arrows in the figure. The maximum of $s^+$ and $s^-$ is reached at point B. It is clear that this model considers the minimization of the input and the maximization of the output in order to reach the effective point on the border which is the furthest from D.

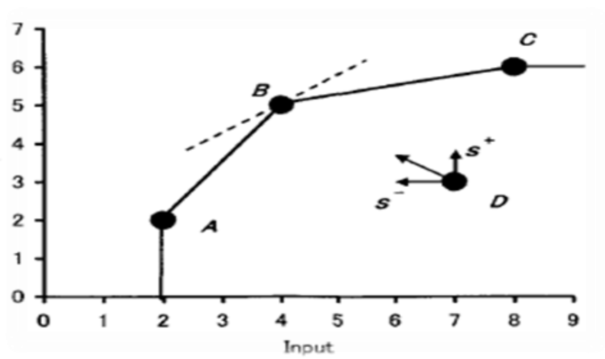


**FIGURE 1: THE ADDITIVE MODEL**

**(William Cooper, Lawrence, Seiford & Kaoru tone (2006))**

The optimal solutions of additive model (1) are provided in the form:$(\gamma^*, s^{+*}, s^{-*})$. It defines efficiency for Decision Making Unit as follows:

- Decision Making Unit is effective in the additive model if and only if all slack variables are zeros$(s^{+*} = s^{-*} = 0)$.

It had been noticed that the efficiency scores $\theta^*$ are not explicitly measured in the additive model eventhough they are present in the slack variables s + and s-. In fact, $\theta^*$only reflects the effectiveness of Farrell, while the purpose of the additive model is to measure all the inefficiencies that can be identified by the model inputs and outputs.

At the optimum and by referring to the model (1), the variables $y^*_{r0}$, $x^*_{i0}$ are presented as follows:

$$\frac{\Delta y_{r0}}{\Delta x_{i0}} = \frac{s^{+*}_{r0}}{s^{-*}_{i0}}$$ = the marginal production of output r to input i. (5)

$$\frac{\Delta y_{r0}}{\Delta x_{k0}} = \frac{s^{+*}_{r0}}{s^{-*}_{k0}}$$ = the marginal rate of transformation between the output r and output k. (6)

$$\frac{\Delta x_{i0}}{\Delta x_{l0}} = \frac{s^{-*}_{i0}}{s^{+*}_{l0}}$$ = the marginal rate of substitution between input i and the input l. (7)

## III. APPLICATION TO THE TUNISIAN CEREAL TECHNOLOGY

The objective of this work is to determine the possibilities of substitution within Tunisian cereal production factors using the non-parametric approach and adopting the additive model. We present here a nonparametric estimation of the marginal rates of substitution according to a data drawn from a survey in Bousalem during the agricultural season 2008-2009. Indeed, it covers an area of 5902.2 hectares is 9% of the surface area of irrigated cereals during the agricultural season 2008-2009. Table 1 shows the distribution of cereal farms to 6 categories of selected surface.

**TABLE 1**
**DISTRIBUTION OF FARMS TO EACH SURFACE CATEGORY**

| Surface category | <5 Ha | 5-10 | 10-20 | 20-50 | 50+ | total |
|---|---|---|---|---|---|---|
| Number of operations | 376 | 286 | 116 | 28 | 10 | 816 |
| % | 46.07 | 35.04 | 14.21 | 3.3 | 1.12 | 100 |
| Total | 46.07 | 81.11 | 95.32 | 98.62 | 100 | ---- |

**(Source : CTV Bousalem)**

The most important feature of this distribution is the dominance of small farms. In fact, 81.11% have less than 10 hectares and 14.21% of the farms of medium sized have more than 10 hectares and less than 20 hectares. The farms of over 50 hectares, on the other hand, represent only 33% of the whole.

The means at our disposal allow us to reach a medium number of operations. So we selected a sample of 97 farmers which represents 11.88% of the sampling rate. The 97 farms were dispersed over the 6 surface layers as follows:

**TABLE 2**
**SAMPLE SIZE**

| Surface category | <5 Ha | 5-10 | 10-20 | 20-50 | 50-100 | total |
|---|---|---|---|---|---|---|
| Number of operations | 37 | 12 | 28 | 10 | 10 | 97 |
| % | 38.14 | 12.37 | 28.86 | 10.3 | 10.3 | 100 |
| Total | 38.14 | 50.51 | 79.37 | 89.67 | 100 | ---- |

This study is in partnership between the research unit AEDD and the National Institute of crops sector. The collected data are drawn from different surveys in selected delegations. The collected data is about land use, production techniques, marketing, prices of inputs and outputs and the socio-economic characteristics of cereal farms during 2008-2009.

As for the output, the analysis of farm production system reveals the importance of cereals (durum wheat, soft wheat and barley) and straws as two separate activities. We provide each farmer respectively with his production of durum wheat, soft wheat and barley expressed in hundredweight, and his sales prices. Then, we will provide the number of straw products and its sales prices.

In summary we will present the outputs of operations by the variables:

**Cereals**

The value of production of durum wheat, soft wheat and barley in Tunisian dinars

**Straws**

The value of straw product in Tunisian dinars

However, the inputs will be presented according to the following variables.

**Land (lan)**

We provide each farmer with a cultivated cereal area. We used the rental price of land as the flow of the use of cereal land.

**Mechanization (Me)**

This aggregate includes the different working operations of the soil and the crop. For each operation, we provide the number of hours and the price paid for each hour of work.

**Labor (lab)**

For each operation, we set out the number of days of family, occasional or wage work labor and the price of a day's work. The family work labor available in one operation may be used in another. To do this, we assign a salary to the family work labor which is equal to the salary of wage work labor.

**Pesticides (pe)**

This aggregate includes all products (herbicide and fungicide) and the market price of each product used.

**Fertilization (fe)**

It consists of fertilizer products (nitrogen, manure and the DAP). The amount of fertilizer product used, expressed as quintal and their market prices are provided.

**Seeds (se)**

We have the amount of seed used, measured in hundredweight, and its market price.

**Water of Irrigation (wa)**

This variable, measured in $m^3$, represents the sum of quantities consumed from the grid.

Thus, the typology from field surveys allowed us to choose the types of operations that may represent homogenous groups. Thereby, we chose two types of operations:

- Operation Type A: These are farms in larger sizes with an agricultural area of more than 20 hectares and which have a diversified cropping system with integration of livestock. Mechanization is pushed with recourse to hire harvesters, employment levels are important and finally labor is permanent and occasional.

- Operation type B: These are small-sized farms with a surface area that is less than 20 hectares. It is a traditional cultural system that is based on cereals and livestock. Mechanization is rented, inputs are used in a quite weak way and manpower is often family labor.

The marginal rate of substitution of the two factors 1 and 2 measures the change along an isoquant in the amount of factor 2 which is needed in order to compensate a too small variation in the amount of factor 1.

We have used the model (1) in order to calculate the marginal rate of substitution between the various inputs. Then, we have proceeded in eliminating the possibility of variations in the production of their adjusted efficiency values to comply with the concepts of isoquants.

The model becomes as follows

$$\text{Max } P^{lan}S_{lan} + P^{mec}S_{mec} + P^{lab}S_{lab} + P^{pe}S_{pe} + P^{fe}S_{fe} + P^{se}S_{se} + P^{wa}S_{wa}$$

$$\text{S/c} \begin{cases} X_0^{lan} = \sum_{j=1}^{N} X_j^{lan}\gamma_j + S_{lan} \\ X_0^{mec} = \sum_{j=1}^{N} X_j^{mec}\gamma_j + S_{mec} \\ X_0^{lab} = \sum_{j=1}^{N} X_j^{lab}\gamma_j + S_{lab} \\ X_0^{pe} = \sum_{j=1}^{N} X_j^{pe}\gamma_j + S_{pe} \\ X_0^{fe} = \sum_{j=1}^{N} X_j^{fe}\gamma_j + S_{fe} \\ X_0^{se} = \sum_{j=1}^{N} X_j^{se}\gamma_j + S_{se} \\ X_0^{wa} = \sum_{j=1}^{N} X_j^{wa}\gamma_j + S_{wa} \\ 1 = \sum_{j=1}^{N}\gamma_j \text{ , } \gamma_j \geq 0 \end{cases} \quad (8)$$

With: $P^i$ denotes the average price of input i defined above

$X_j^i$ represents the demand for input i by the operation j

$S_i$ is a variable of input slack i

$\gamma_j$ is an intensity variable

N is the number of farms.

As previously sets the marginal rate of substitution (MRS) between the factors i and j is calculated as follows:

$$MRS_j^i = \frac{\Delta x_i}{\Delta x_j} = \frac{s_i^{-*}}{s_j^{-*}}$$

To measure the impact of changes of this rate over the productive combination, we used what is called the elasticity of substitution. In fact, this concept is jointly introduced by Hicks (1932) and Robinson (1933): it measures, for a given level of production, the impact of variation of the marginal productivity over the ratio of two factors of production. The calculation of this elasticity requires the assumption that is related to the firmness of other factors of production.

The elasticity of substitution of the production factors in is measured, a nonparametric approach, by Cooper Park and Pastor (2000) through the additive model as follows:

$$\sigma_j^i = \frac{\frac{\Delta x_i}{x_i}}{\frac{\Delta x_j}{x_j}} = \frac{x_j}{x_i}\frac{\Delta x_i}{\Delta x_j} = \frac{x_j}{x_i}\frac{s_i^{-*}}{s_j^{-*}} = TMS_j^i\,\frac{x_j}{x_i} = ES_j^i$$

In general, if there is large substitutability of inputs, the elasticity of substitution will be even higher. A positive value of the elasticity of substitution reflects the possibilities of substitution between production factors.

At the optimum, the model (8) applied to the data of the Delegation of Bousalem presents the following solutions:

| | $S_{lan}^-$ | $S_{mec}^-$ | $S_{lab}^-$ | $S_{pe}^-$ | $S_{fe}^-$ | $S_{se}^-$ | $S_{wa}^-$ |
|---|---|---|---|---|---|---|---|
| Type A | 10 | 5 | 10 | -20 | -20 | -50 | 15 |
| Type B | 12 | 7 | 9 | -21 | -18 | -23 | 12,5 |

The following table presents the marginal rate of technical substitution estimated for productions factors of the delegation of Bousalem.

| | Type A | Type B |
|---|---|---|
| $TMST_{lan}^{wa}$ | **1.5** | **1.04** |
| $TMST_{mec}^{wa}$ | **3** | **1.74** |
| $TMST_{lab}^{wa}$ | **1.5** | **1.384** |
| $TMST_{pe}^{wa}$ | -0.75 | -0.595 |
| $TMST_{fe}^{wa}$ | -0.75 | -0.694 |
| $TMST_{se}^{wa}$ | -0.3 | -0.543 |
| $TMST_{mec}^{lan}$ | **2** | **1.71** |
| $TMST_{lab}^{lan}$ | **1** | **1.33** |
| $TMST_{pe}^{lan}$ | -0.5 | -0.574 |
| $TMST_{fe}^{lan}$ | -0.5 | -0.666 |
| $TMST_{se}^{terre}$ | -0.2 | -0.521 |
| $TMST_{lab}^{mec}$ | **0.5** | **0.77** |
| $TMST_{pe}^{mec}$ | -0.25 | -0.33 |
| $TMST_{fe}^{mec}$ | -0.25 | -0.38 |
| $TMST_{se}^{mec}$ | -0.1 | -0.30 |
| $TMST_{pe}^{lab}$ | -0.5 | -042 |
| $TMST_{fe}^{lab}$ | -0.5 | -0.5 |
| $TMST_{se}^{lab}$ | -0.2 | -0.39 |
| $TMST_{fe}^{pe}$ | -1 | **1.16** |
| $TMST_{se}^{pe}$ | -0.4 | **0.913** |
| $TMST_{se}^{fe}$ | **0.4** | **0.782** |

The substitutability exists between the earth and the following factors of production: mechanization and labor along with a MRS that is much higher than the unit. Substitutability between mechanization and labor, on the other hand, is low along with a MRS that is lower than the Unit

The table above shows that the MRS is higher than the unit between water and land, between water and mechanization, between water and labor, between land and mechanization and between the earth and labor.

The best substitute for holdings of the delegation of Bousalem is that between water and mechanization with a $MRS_{mec-}^{wa}$ that is equal to 3 for Type A operations and equal to 1.74 for Type B operations.

This substitution allows the technically efficient grain to reduce the cost of the production of a hectare of cereal by 56.18 dinars that is to say 3.47% and thus improves the allocative efficiency for Type A. This substitution for farms of type B on the other hand, reduces the cost of production of a hectare of cereal by 80.91 dinars that is to say 5.52%.

## IV. CONCLUSION

The measure of substitutability of cereal production factors in the delegation of Bousalem allows us to determine the existence of substitutability between different factors of production.

Substitution exists between land and other production factors and between mechanization and other inputs except labor. The estimated elasticities of substitution between irrigation water and other factors of production of the delegation of Bousalem allow us to conclude that the irrigation water and labor are substitutable.

Results were found relative to the selected delegation and cannot be extrapolated even though some recommendations seem valid for other delegations such as the technology used. The credibility of the data collected remains related to the fairness and the reliability of the grain producers' declarations.

Although we have tested the reliability of our model, the approach used for measuring the level of effectiveness embodies conceptual limits (deterministic approach) and operational limits (sensitivity to changing variables). We also note that we have presented the possibilities of substitution of grain farms inputs relating to operating conditions of a single crop (2008-2009), and then it would be wiser to consider at least three different campaigns to investigate the impact of climatic conditions and agricultural and economic policies on the behavior of grain farmers. An extension of this work would be to look for the external factors that affect the production efficiency in order to better understand the real situation of cereal farms.

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